Calculus 1 Test 3 — Summer 2013

NAME KEY STUDENT NUMBER ______________

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Use equal signs when things are equal (and don't use them when things are not equal). Indicate when you have used L'Hopital's Rule, as we did in class. If applicable, put your answer in the box provided. Each numbered problem is worth 10 points. You may use your calculator any way you see fit, but **you may not share calculators**!!!

p261 #13

1. (a) Evaluate $\lim_{t\to 0} \frac{\sin t^2}{t}$.

$$\lim_{t\to 0} \frac{\sin(t^2)}{t} \overset{0/0}{=} \lim_{t\to 0} \frac{\cos(t^2)[2t]}{1} = \frac{\cos(0^2)(2\cdot 0)}{1} = 0$$

$\boxed{0}$

p261 #21

(b) Evaluate $\lim_{x\to 0} \frac{x^2}{\ln(\sec x)}$.

$$\lim_{x\to 0} \frac{x^2}{\ln(\sec(x))} \overset{0/0}{=} \lim_{x\to 0} \frac{2x}{\frac{1}{\sec(x)}[\sec(x)\tan(x)]} = \lim_{x\to 0} \frac{2x}{\tan(x)}$$

$$\overset{0/0}{=} \lim_{x\to 0} \frac{2}{\sec^2(x)} = \frac{2}{\sec^2(0)} = \frac{2}{(1)^2} = 2$$

$\boxed{2}$

2. Evaluate $\lim_{x\to\infty} (\ln x)^{1/x}$.

p261 #53

Let $y = (\ln x)^{1/x}$ and so $\ln(y) = \ln((\ln x)^{1/x}) = \frac{1}{x}(\ln(\ln x))$.

So $\lim_{x\to\infty} (\ln y) = \lim_{x\to\infty} \frac{\ln(\ln x)}{x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}[\frac{1}{x}]}{1} = \lim_{x\to\infty} \frac{1}{x}\frac{1}{\ln x} = 0.$

So $e^{\lim_{x\to\infty}(\ln y)} = e^0$ or $\lim_{x\to\infty}(e^{\ln y}) = 1$ since $\exp(x)$ is continuous

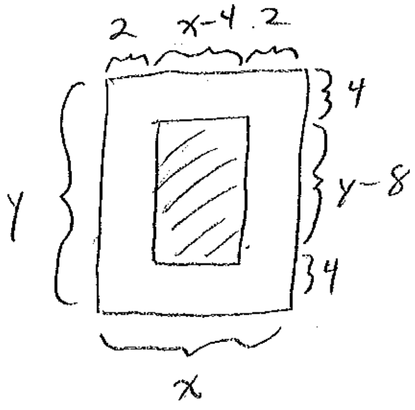
or $\lim_{x\to\infty}(y) = \lim_{x\to\infty}(\ln x)^{1/x} = 1$

$\boxed{1}$

p269 #11

3,4. You are designing a rectangular poster to contain 50 in^2 of printing with a 4 in. margin at the top and bottom and a 2 in. margin at each side. What overall dimensions will minimize the amount of paper used? Show all steps and confirm that your answer really is a minimum.

(1)



(2) The question is the MIN area $A = xy$.

(3) We know that printed area is $(x-4)(y-8)=50$

or $y = \frac{50}{x-4} + 8$.

(4) Area is

$$A = xy = x\left(\frac{50}{x-4}+8\right) = A(x)$$

(5) MIN $A(x) = \frac{50x}{x-4} + 8x$ for $x \in (4, \infty)$.

Well, $A'(x) = \frac{[50](x-4)-(50x)[1]}{(x-4)^2} + 8 = \frac{-200}{(x-4)^2} + 8$.

Set $A'(x) = \frac{-200}{(x-4)^2} + 8 \equiv 0 \Rightarrow (x-4)^2 = 25 \Rightarrow x-4 = \pm 5$

$\Rightarrow x = -1$ and $x = 9$ are critical points.

Now, $A''(x) = \frac{400}{(x-4)^3}$ and $A''(9) = \frac{400}{5^3} = \frac{400}{125} > 0$.

So by the Second Derivative Test, ++

A has a MIN at $x = 9$ in, $y = 18$ in.

$\boxed{x = 9 \text{ in}, \ y = 18 \text{ in}}$

OR consider

	$(4,9)$	$(9,\infty)$
k	5	14
$A'(k)$	-192	6
$A'(x)$	−	+
$A(x)$	DEC	INC

So by the First Derivative Test, A has a MIN at $x = 9$ in, $y = 18$ in

5. Use Newton's method to estimate the one real solution of $x^3 + 3x + 1 = 0$. Start with $x_0 = 0$ and then find x_2.

p277 #2

Well, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$. We have $f(x) = x^3 + 3x + 1$ and $f'(x) = 3x^2 + 3$.

So $x_1 = 0 - \dfrac{f(0)}{f'(0)} = -\dfrac{1}{3}$, and

$$x_2 = \left(-\frac{1}{3}\right) - \frac{f(-1/3)}{f'(-1/3)} = -\frac{1}{3} - \frac{\left(-\frac{1}{3}\right)^3 + 3\left(-\frac{1}{3}\right) + 1}{3\left(-\frac{1}{3}\right)^2 + 3} = -\frac{1}{3} + \frac{1}{90} = \frac{-29}{90}$$

$$\boxed{x_2 = \frac{-29}{90}}$$

6. Solve the initial value problem: $\dfrac{dv}{dt} = \dfrac{8}{1+t^2} + \sec^2 t,\ v(0) = 1.$

p287 #104

$v(t) \in \int \left(\frac{dv}{dt}\right) dt = \int \left(\frac{8}{1+t^2} + \sec^2(t)\right) dt = 8\tan^{-1}(t) + \tan(t) + C.$

So $v(t) = 8\tan^{-1}(t) + \tan(t) + k$ for some k. Now

$v(0) = 8\tan^{-1}(0) + \tan(0) + k \equiv 1 \Rightarrow k = 1.$

So $v(t) = 8\tan^{-1}(t) + \tan(t) + 1$

$$v(t) = \boxed{8\tan^{-1}(t) + \tan(t) + 1}$$

p322 #10c

7. Suppose f and h are integrable and that

$$\int_1^9 f(x)\,dx = -1, \qquad \int_7^9 f(x)\,dx = 5, \qquad \int_7^9 h(x)\,dx = 4.$$

Evaluate $\int_7^9 (2f(x) - 3h(x))\,dx$.

Well, $\int_7^9 (2f(x) - 3h(x))\,dx = 2\int_7^9 f(x)\,dx - 3\int_7^9 h(x)\,dx$

$= 2(5) - 3(4) = 10 - 12 = -2$

$$\boxed{-2}$$

p327, 328

8. State the Fundamental Theorem of Calculus (both parts, with hypotheses).

① If f is continuous on $[a, b]$ then $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point of $[a,b]$ and

$$\frac{dF}{dx} = \frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

② If f is continuous at every point of $[a,b]$ and if F is any antiderivative of f on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

p342 #11

9. (a) Evaluate $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}$

let $u = 1 - r^3$

$du = -3r^2\,dr$

$\frac{du}{-3} = r^2\,dr$

$$= \int \frac{9}{\sqrt{u}} \frac{du}{-3} = -3\int u^{-1/2}\,du = -3\left(2u^{1/2}\right) + C = -6\sqrt{u} + C$$

$$\boxed{= -6\sqrt{1-r^3} + C}$$

p343 #54

TYPO! → $e^{1/x}$

(b) Evaluate $\int \frac{1}{x^2} \sec(1 + e^{1/x}) \tan(1 + e^{1/x})\,dx.$

let $u = 1 + e^{1/x}$

$du = e^{1/x}\left[-\frac{1}{x^2}\right] dx$

$-du = \frac{1}{x^2} e^{1/x}\,dx$

$$= \int \sec(u)\tan(u)(-du) = -\sec(u) + C$$

$$= -\sec\left(1 + e^{1/x}\right) + C$$

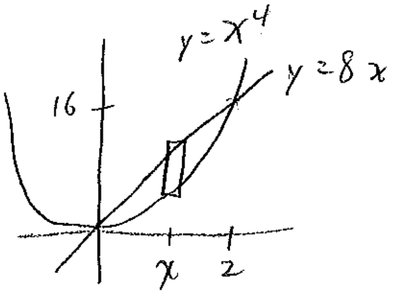
$$\boxed{-\sec(1 + e^{1/x}) + C}$$

10. Find the area of the region enclosed by the curves $y = x^4$ and $y = 8x$.

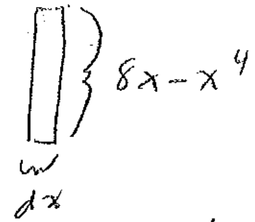
p352 #65

These curves intersect when $x^4 = 8x \Rightarrow x = 0$ or $x = 2$.

So:



For a dx-slice:



Area of slice is $(8x - x^4)\,dx$.

So total area is $A = \int_0^2 (8x - x^4)\,dx = \left(4x^2 - \frac{1}{5}x^5\right)\Big|_0^2$

$= \left(16 - \frac{32}{5}\right) - 0 = \frac{80}{5} - \frac{32}{5} = \frac{48}{5}$

$\boxed{\frac{48}{5}}$

Bonus. If av(f) really is a typical value of the integrable function $f(x)$ on $[a, b]$, then the constant function av(f) should have the same integral over $[a, b]$ as f. Does it? That is, does

p323 #81

$$\int_a^b \text{av}(f)\,dx = \int_a^b f(x)\,dx?$$

Recall $\text{av}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$, or $\text{av}(f)(b-a) = \int_a^b f(x)\,dx$.

So $\int_a^b \text{av}(f)\,dx = \text{av}(f)\,x\Big|_a^b = \text{av}(f)\,(b-a)$.

So YES, $\text{av}(f)\,(b-a) = \int_a^b \text{av}(f)\,dx = \int_a^b f(x)\,dx$.